TENTH EDITION

BASIC TECHNICAL MATHEMATICS with Calculus

SI VERSION

ALLYN J. WASHINGTON • MICHELLE BOUÉ

Basic Technical Mathematics with Calculus

SI Version

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TENTH EDITION

Basic Technical Mathematics with Calculus

SI Version

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To Douglas, Julia and Andrea ~Michelle Boué

In memory of my loving wife, Millie ~Allyn J. Washington

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Preface

Scope of the Book

Basic Technical Mathematics with Calculus, SI Version, tenth edition, is intended primarily for students in technical and pre-engineering technology programs or other programs for which coverage of basic mathematics is required. Chapters 1 through 20 provide the necessary background for further study, with an integrated treatment of algebra and trigonometry. Chapter 21 covers the basic topics of analytic geometry, and Chapter 22 gives an introduction to statistics. Fundamental topics of calculus are covered in Chapters 23 through 31. In the examples and exercises, numerous applications from many fields of technology are included, primarily to indicate where and how mathematical techniques are used. However, it is not necessary that the student have a specific knowledge of the technical area from which any given problem is taken.

Most students using this text will have a background that includes some algebra and geometry. However, the material is presented in adequate detail for those who may need more study in these areas. The material presented here is sufficient for three to four semesters.

One of the principal reasons for the arrangement of topics in this text is to present material in an order that allows a student to take courses concurrently in allied technical areas, such as physics and electricity. These allied courses normally require a student to know certain mathematical topics by certain definite times; yet the traditional order of topics in mathematics courses makes it difficult to attain this coverage without loss of continuity. However, the material in this book can be rearranged to fit any appropriate sequence of topics. Another feature of this text is that certain topics traditionally included for mathematical completeness have been covered only briefly or have been omitted. The approach used in this text is not unduly rigorous mathematically, although all appropriate terms and concepts are introduced as needed and given an intuitive or algebraic foundation. The aim is to help the student develop an understanding of mathematical methods without simply providing a collection of formulas. The text material has been developed with the recognition that it is essential for the student to have a sound background in algebra and trigonometry in order to understand and succeed in any subsequent work in mathematics.

New Features

In this tenth edition of *Basic Technical Mathematics with Calculus, SI Version*, we have retained all the basic features of successful previous editions and have also introduced a number of improvements, described here.

NEW AND REVISED COVERAGE

The topics of units and measurement covered in an appendix in the ninth edition have been expanded and integrated into Chapter 1, together with new discussions on rounding and on engineering notation. Interval notation is introduced in Chapter 3 and is used in several sections throughout the text. Chapter 31 includes a new subsection on solving nonhomogeneous differential equations using Fourier series.

Chapter 22 has been revised and expanded; a new section on summarizing data covers measures of central tendency, measures of spread, and new material on Chebychev's theorem; the section on normal distributions now includes a subsection on sampling distributions. In addition, the chapter now includes a completely new section on confidence intervals.

EXPANDED PEDAGOGY

- NEW "Common Error" boxes appear throughout the text. A fresh design emphasizes valuable warnings against common mistakes or areas where students frequently have difficulty. These boxes replace the notes flagged by a "Caution" indicator in the previous edition.

- NEW "Learning Tip" boxes appear in the margin throughout the text. These colourful boxes highlight the underlying rationale of using specific mathematical functions and encourage students to think strategically about how and why specific mathematical concepts are needed and applied. They also focus attention on material that is of particular importance in understanding the topic under discussion. These boxes replace the notes flagged by a "Notes" indicator in the previous edition.
- NEW "Procedure" boxes include step-by-step instructions on how to perform select calculations.

FEWER CALCULATOR SCREENS

Many figures involving screens from a graphic calculator have been either removed from the text or replaced by regular graphs. The calculator displays that remain are, for the most part, related to topics that require the use of technology (such as the graphical solution of systems of equations) or topics where technology can greatly simplify a process (such as obtaining the inverse of a large matrix). The appendix on graphing calculators from the previous edition dedicated to the graphing calculator will be available in Chapter 34 of the Study Plan in both MyMathLab and MathXL versions of this course. Students will also have easy access to it through the eText in MyMathLab.

FUNCTIONAL USE OF COLOUR

The new full-colour design of this edition uses colour effectively for didactical purposes. Many figures and graphs have been enhanced with colour. Moreover, colour is used to identify and focus attention on the text's new pedagogical features. Colour is also used to highlight the question numbers of writing exercises so that students and instructors can identify them easily.

NOTATION

Symbols used in accordance with professional Canadian standards are applied consistently throughout the text.

INCREASED BREADTH OF APPLICATIONS

New examples and exercises have been added in order to increase the range of applications covered by the text. New material can be found involving statics, fluid mechanics, optics, acoustics, cryptography, forestry, reliability, and quality control, to name but a few.

INTERNATIONAL AND CANADIAN CONTENT

New Canadian content appears either in the form of examples within the text (some of which are linked to chapter openers, so they are accompanied by a full colour image), or as exercises at the end of a section or chapter. All material of global interest has been retained or updated, and some new exercises were also added.

LEARNING OUTCOMES

A list of Learning Outcomes appears on the introductory page of each chapter, replacing the list of key topics for each section in the previous edition. This new learning tool reflects the current emphasis on learning outcomes and gives the student and instructor a quick way of checking that they have covered key contents of the chapter.

Continuing Features

EXAMPLE DESCRIPTIONS

A brief descriptive title is given with each example number. This gives an easy reference for the example, which is particularly helpful when a student is reviewing the contents of the section.

PRACTICE EXERCISES

Throughout the text, there are *practice exercises* in the margin. Most sections have at least one (and up to as many as four) of these basic exercises. They are included so that a student is more actively involved in the learning process and can check his or her understanding of the material to that point in the section. They can also be used for classroom exercises. The answers to these exercises are given at the end of the exercise set for the section.

NEW EXERCISES

More than 300 exercises are new or have been updated. This tenth edition contains a total of about 12 500 exercises.

CHAPTER INTRODUCTIONS

Each chapter introduction illustrates specific examples of how the development of technology has been related to the development of mathematics. These introductions show that past discoveries in technology led to some of the methods in mathematics, whereas in other cases mathematical topics already known were later very useful in bringing about advances in technology.

SPECIAL EXPLANATORY COMMENTS

Throughout the book, special explanatory comments in colour have been used in the examples to emphasize and clarify certain important points. Arrows are often used to indicate clearly the part of the example to which reference is made.

IMPORTANT FORMULAS

Throughout the book, important formulas are set off and displayed so that they can be easily referenced.

SUBHEADS AND KEY TERMS

Many sections include subheads to indicate where the discussion of a new topic starts within the section. Key terms are noted in the margin for emphasis and easy reference.

EXERCISES DIRECTLY REFERENCED TO TEXT EXAMPLES

The first few exercises in most of the text sections are referenced directly to a specific example of the section. These exercises are worded so that it is necessary for the student to refer to the example in order to complete the required solution. In this way, the student should be able to review and understand the text material better before attempting to solve the exercises that follow.

WRITING EXERCISES

One specific writing exercise is included at the end of each chapter. These exercises give the students practice in explaining their solutions. Also, there are more than 400 additional exercises throughout the book (at least 8 in each chapter) that require at least a sentence or two of explanation as part of the answer. The question numbers of writing exercises are highlighted in colour. A special "Index of Writing Exercises" is included at the back of the book.

WORD PROBLEMS

There are more than 120 examples throughout the text that show the complete solutions of word problems. There are also more than 850 exercises in which word problems are to be solved.

CHAPTER EQUATIONS, REVIEW EXERCISES, AND PRACTICE TESTS

At the end of each chapter, all important equations are listed together for easy reference. Each chapter is also followed by a set of review exercises that covers all the material in the chapter. Following the chapter equations and review exercises is a chapter practice test that students can use to check their understanding of the material. Solutions to all practice test problems are given in the back of the book.

APPLICATIONS

Examples and exercises illustrate the application of mathematics in all fields of technology. Many relate to modern technology such as computer design, electronics, solar energy, lasers, fibre optics, the environment, and space technology. A special "Index of Applications" is included near the end of the book.

EXAMPLES

There are more than 1400 worked examples in this text. Of these, more than 300 illustrate technical applications.

FIGURES

There are more than 1300 figures in the text. Approximately 20% of the figures are new or have been modified for this edition.

MARGIN NOTES

Throughout the text, some margin notes briefly point out relevant historical events in mathematics and technology. Other margin notes are used to make specific comments related to the text material. Also, where appropriate, equations from earlier material are shown for reference in the margin.

ANSWERS TO EXERCISES

The answers to all odd-numbered exercises (except the end-of-chapter writing exercises) are given at the back of the book.

FLEXIBILITY OF MATERIAL COVERAGE

The order of material coverage can be changed in many places, and certain sections may be omitted without loss of continuity of coverage. Users of earlier editions have indicated the successful use of numerous variations in coverage. Any changes will depend on the type of course and completeness required.

Supplements

SUPPLEMENTS FOR THE STUDENT

Extensively updated by text author Michelle Boué, the *Students Solutions Manual* contains revised solutions for every other odd-numbered exercise. These step-by-step solutions have been expanded for even greater accuracy, clarity, and consistency to improve student problem-solving skills. The *Students Solutions Manual* is included in MyMathLab and is also available as a printed supplement via the Pearson Custom Library. (Please contact your local Pearson representative to learn more about this option.)

SUPPLEMENTS FOR THE INSTRUCTOR

Instructor's resources include the following supplements.

Instructor's Solutions Manual

The *Instructor's Solution Manual* contains detailed solutions to every section exercise, including review exercises. These in-depth, step-by-step solutions have been thoroughly revised by text author Michelle Boué for greater clarity and consistency; note that this expansion has been carried through to the Student Solutions Manual as well. The *Instructors Solutions Manual* can be downloaded from Pearson's online catalogue at www.pearsoned.ca. The *Instructor's Solution Manual* contains solutions for all section exercises.

Animated PowerPoint Presentations

More than 150 animated slides are available for download from a protected location on Pearson Education's online catalogue, at www.pearsoned.ca.

Each slide offers a step-by-step mini lesson on an individual section, or key concept, formula, or equation from the first 28 chapters of the book. For instance, 15 steps for using the "General Power Formula for Integration" are beautifully illustrated in the animated slide for Chapter 28. There are two sets of slides for "Operations with Complex Numbers" for section 2 of Chapter 12; the 9 steps to perform addition are shown on one slide, and the 13 steps to perform subtraction appear on the second slide.

These animated slides offer bite-sized chunks of key information for students to review and process prior to going to the homework questions for practice. Please note that not every section in every chapter is accompanied by an animated slide as some topics lend themselves to this approach more than others. These PowerPoint slide are also integrated in the Pearson eText within MyMathLab.

TestGen with Algorithmically Generated Questions

Instructors can easily create tests from textbook section objectives. Algorithmically generated questions allow unlimited versions. Instructors can edit problems or create their own by using the built-in question editor to generate graphs; import graphics; and insert math notation, variable numbers, or text. Tests can be printed or administered online via the Web or other network.

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MyMathLab delivers **proven results** in helping individual students succeed:

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MyMathLab provides **engaging experiences** that personalize, stimulate, and measure learning for each student:

- **Exercises:** The homework and practice exercises in MyMathLab are correlated to the exercises in the textbook, and they regenerate algorithmically to give students unlimited opportunity for practice and mastery. The software offers immediate, helpful feedback when students enter incorrect answers.
- **Multimedia learning aids:** Exercises include guided solutions, sample problems, animations, videos, and eText clips for extra help at point-of-use.
- **Expert tutoring:** Although many students describe the whole of MyMathLab as "like having your own personal tutor," students using MyMathLab do have access to live tutoring from Pearson, from qualified mathematics and statistics instructors who provide tutoring sessions for students via MyMathLab.

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To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit **www.mymathlab.com** or contact your Pearson representative.

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Basic Algebraic Operations

1

Interest in things such as the land on which they lived, the structures they built, and the motion of the planets led people in early civilizations to keep records and to create methods of counting and measuring. In turn, some of the early ideas of arithmetic, geometry, and trigonometry were developed. From such beginnings, mathematics has played a key role in the great advances in science and technology.

Often, mathematical methods were developed from studies made in sciences, such as astronomy and physics, to better describe, measure, and understand the subject being studied. Some of these methods resulted from the needs in a particular area of application.

Many people were interested in the mathematics itself and added to what was then known. Although this additional mathematical knowledge may not have been related to applications at the time it was developed, it often later became useful in applied areas.

In the chapter introductions that follow, examples of the interaction of technology and mathematics are given. From these examples and the text material, it is hoped you will better understand the important role that mathematics has had and still has in technology. In this text, there are applications from technologies including (but not limited to) aeronautical, business, communications, electricity, electronics, engineering, environmental, heat and air conditioning, mechanical, medical, meteorology, petroleum, product design, solar, and space. To solve the applied problems in this text will require a knowledge of the mathematics presented but will *not* require prior knowledge of the field of application.

We begin by reviewing the concepts that deal with numbers and symbols. This will enable us to develop topics in algebra, an understanding of which is essential for progress in other areas such as geometry, trigonometry, and calculus.

▶ The Great Pyramid at Giza in Egypt was built about 4500 years ago.

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◀ In the 1500s, 1600s, and 1700s, discoveries in astronomy and the need for more accurate maps and instruments in navigation were very important in leading scientists and mathematicians to develop useful new ideas and methods in mathematics.

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LEARNING OUTCOMES

After completion of this chapter, the student should be able to:

- Identify real, imaginary, rational, and irrational numbers
- Perform mathematical operations on integers, decimals, fractions, and radicals
- Use the fundamental laws of algebra in numeric and algebraic equations
- Employ mathematical order of operations
- Understand technical measurement and approximation, as well as the use of significant digits and rounding
- Use scientific and engineering notations
- Convert units of measurement
- Rearrange and solve basic algebraic expressions
- Interpret word problems using algebraic symbols

◀ Late in the 1800s, scientists were studying the nature of light. This led to a mathematical prediction of the existence of radio waves, now used in many types of communication. Also, in the 1900s and 2000s, mathematics has been vital to the development of electronics and space travel.

1.1 Numbers

Real Number System • Number Line • Absolute Value • Signs of Inequality • Reciprocal • Denominate Numbers • Literal Numbers

In technology and science, as well as in everyday life, we use the very familiar **counting numbers** 1, 2, 3, and so on. They are also called **natural numbers** or **positive integers.** The **negative integers** $-1, -2, -3$, and so on are also very necessary and useful in mathematics and its applications. *The* **integers** *include the positive integers and the negative integers and* **zero,** *which is neither positive nor negative*. This means the integers are the numbers . . . , $-3, -2, -1, 0, 1, 2, 3$, and so on.

To specify parts of a quantity, *rational numbers* are used. *A* **rational number** *is any number that can be represented by the division of one integer by another nonzero integer*. Another type of number, an **irrational number,** *cannot be written as the division of one integer by another.*

■ Irrational numbers were discussed by the Greek mathematician Pythagoras in about 540 B.C.E.

EXAMPLE 1 Identifying rational numbers and irrational numbers

The numbers 5 and -19 are integers. They are also rational numbers since they can be written as $\frac{5}{1}$ and $\frac{-19}{1}$, respectively. Normally, we do not write the 1's in the denominators.

The numbers $\frac{5}{8}$ and $\frac{-11}{3}$ are rational numbers because the numerator and the denominator of each are integers.

The numbers $\sqrt{2}$ and π are irrational numbers. It is not possible to find two integers, one divided by the other, to represent either of these numbers. It can be shown that square roots (and other roots) that cannot be expressed exactly in decimal form are irrational. Also, $\frac{22}{7}$ is sometimes used as an *approximation* for π, but it is not equal *exactly* to π. We must remember that $\frac{22}{7}$ is rational and π is irrational.

The decimal number 1.5 is rational since it can be written as $\frac{3}{2}$. Any such *terminating decimal* is rational. The number 0.6666 . . . , where the 6's continue on indefinitely, is rational since we may write it as $\frac{2}{3}$. In fact, any *repeating decimal* (in decimal form, a specific sequence of digits is repeated indefinitely) is rational. The decimal number 0.673 273 273 2 . . . is a repeating decimal where the sequence of digits 732 is repeated indefinitely $\left(0.673\,273\,273\,2\ldots = \frac{1121}{1665}\right)$. ■

LEARNING TIP

A notation that is often used for repeating decimals is to place a bar over the digits that repeat. Using this notation we can write $\frac{1121}{1665} = 0.6\overline{732}$ and $\frac{2}{3} = 0.\overline{6}$

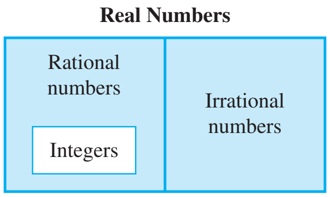

Fig. 1.1

The integers, the rational numbers, and the irrational numbers, including all such numbers that are positive, negative, or zero, make up the **real number system** (see Fig. 1.1). There are times we will encounter an **imaginary number,** *the name given to the square root of a negative number*. Imaginary numbers are not real numbers and will be discussed in Chapter 12. However, unless specifically noted, we will use real numbers. Until Chapter 12, it will be necessary to only *recognize* imaginary numbers when they occur.

Also in Chapter 12, we will consider **complex numbers,** which include both the real numbers and imaginary numbers. See Exercise 37 of this section.

EXAMPLE 2 Identifying real numbers and imaginary numbers

The number 7 is an integer. It is also rational since $7 = \frac{7}{1}$, and it is a real number since the real numbers include all the rational numbers.

The number 3π is irrational, and it is real since the real numbers include all the irrational numbers.

The numbers $\sqrt{-10}$ and $-\sqrt{-7}$ are imaginary numbers.

The number $\frac{-3}{7}$ is rational and real. The number $-\sqrt{7}$ is irrational and real.

The number $\frac{\pi}{6}$ is irrational and real. The number $\frac{\sqrt{-3}}{2}$ is imaginary. ■

■ Real numbers and imaginary numbers are both included in the *complex number system*. See Exercise 37.

A fraction *may contain any number or symbol representing a number in its numerator or in its denominator*. The fraction indicates the division of the numerator by the denominator, as we previously indicated in writing rational numbers. Therefore, a fraction may be a number that is rational, irrational, or imaginary. A fraction can represent

■ Fractions were used by early Egyptians and Babylonians. They were used for calculations that involved parts of measurements, property, and possessions.

a part of a whole, and sometimes it can represent the number of equal-sized parts that a whole is divided into. For example, in Fig. 1.2, a whole circle has been divided into eight equal pieces. The shaded portion represents five of those eight pieces, or 5/8 of the whole circle.

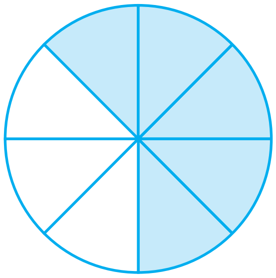

Fig. 1.2

EXAMPLE 3 Fractions

The numbers $\frac{2}{7}$ and $\frac{-3}{2}$ are fractions, and they are rational.

The numbers $\frac{\sqrt{2}}{9}$ and $\frac{6}{\pi}$ are fractions, but they are not rational numbers. It is not possible to express either as one integer divided by another integer.

The number $\frac{\sqrt{-5}}{6}$ is a fraction, and it is an imaginary number. ■

The Number Line

Real numbers may be represented by points on a line. We draw a horizontal line and designate some point on it by O, which we call the **origin** (see Fig. 1.3). The integer *zero* is located at this point. Equal intervals are marked to the right of the origin, and the positive integers are placed at these positions. The other positive rational numbers are located between the integers. The points that cannot be defined as rational numbers represent irrational numbers. We cannot tell whether a given point represents a rational number or an irrational number unless it is specifically marked to indicate its value.

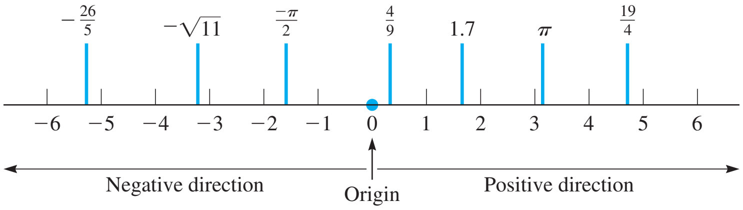

Fig. 1.3

The negative numbers are located on the number line by starting at the origin and marking off equal intervals *to the left, which is the* **negative direction.** As shown in Fig. 1.3, *the positive numbers are to the right of the origin and the negative numbers are to the left of the origin*. Representing numbers in this way is especially useful for graphical methods.

We next define another important concept of a number. The **absolute value** of a number is the numerical value (magnitude) of the number without regard to its sign. The absolute value of a positive number is the number itself, and the absolute value of a negative number is just the number, without the negative sign. On the number line, we may interpret the absolute value of a number as the distance (which is always positive) between the origin and the number. Absolute value is denoted by writing the number between vertical lines, as shown in the following example.

Practice Exercises

1. $|-4.2| = ?$ **2.** $-\left|-\frac{3}{4}\right| = ?$

EXAMPLE 4 Absolute value

The absolute value of 6 is 6, and the absolute value of −7 is 7. We write these as $|6| = 6$ and $|-7| = 7$. See Fig. 1.4.

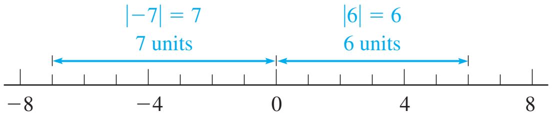

Fig. 1.4

Other examples are $\left|\frac{7}{5}\right| = \frac{7}{5}$, $|-\sqrt{2}| = \sqrt{2}$, $|0| = 0$, $-|\pi| = -\pi$, $|-5.29| = 5.29$, $-|-9| = -9$ since $|-9| = 9$. ■

■ The symbols $=$, $<$, and $>$ were introduced by English mathematicians in the late 1500s.

On the number line, *if a first number is to the right of a second number, then the first number is said to be* **greater than** *the second. If the first number is to the left of the second, it is* **less than** *the second number*. The symbol $>$ designates "is greater than," and the symbol $<$ designates "is less than." These are called **signs of inequality.** See Fig. 1.5.

Practice Exercises

Place the correct sign of inequality ($<$ or $>$) between the given numbers.

3. $-5 \quad 4$ **4.** $0 \quad -3$

EXAMPLE 5 Signs of inequality

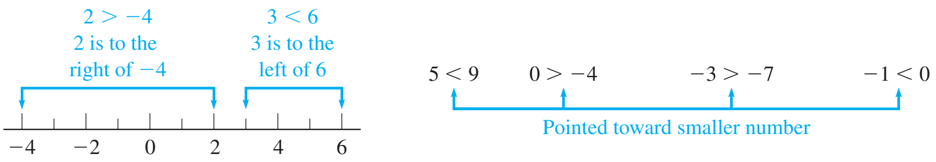

Fig. 1.5 ■

Every number, except zero, has a **reciprocal.** *The reciprocal of a number is 1 divided by the number.*

EXAMPLE 6 Reciprocal

The reciprocal of 7 is $\frac{1}{7}$. The reciprocal of $\frac{2}{3}$ is

$$\frac{1}{\frac{2}{3}} = 1 \times \frac{3}{2} = \frac{3}{2}$$

invert denominator and multiply (from arithmetic)

The reciprocal of 0.5 is $\frac{1}{0.5} = 2$. The reciprocal of $-\pi$ is $-\frac{1}{\pi}$. Note that the negative sign is retained in the reciprocal of a negative number.

We showed the multiplication of 1 and $\frac{3}{2}$ as $1 \times \frac{3}{2}$. We could also show it as $1 \cdot \frac{3}{2}$ or $1\left(\frac{3}{2}\right)$. We will often find the form with parentheses is preferable. ■

In applications, *numbers that represent a measurement and are written with units of measurement are called* **denominate numbers.** The next example illustrates the use of units and the symbols that represent them.

EXAMPLE 7 Denominate numbers

■ For reference, see Section 1.3 for units of measurement and the symbols used for them.

To show that a certain HDTV set has mass of 28 kilograms, we write the mass as 28 kg.

To show that a giant redwood tree is 110 metres high, we write the height as 110 m.

To show that the speed of a rocket is 1500 metres per second, we write the speed as 1500 m/s. (Note the use of s for second. We use s rather than sec.)

To show that the area of a computer chip is 0.75 square centimetres, we write the area as 0.75 cm^2. (We will not use sq cm.)

To show that the volume of water in a glass tube is 25 cubic centimetres, we write the volume as 25 cm^3. (We will not use cu cm or cc.) ■

Literal Numbers

It is usually more convenient to state definitions and operations on numbers in a general form. *To do this, we represent the numbers by letters, called* **literal numbers.** For example, if we want to say "If a first number is to the right of a second number on the number line, then the first number is greater than the second number," we can write "If a is to the right of b on the number line, then $a > b$." Another example of using a literal number is "The reciprocal of n is $1/n$."

Certain literal numbers may take on any allowable value, whereas other literal numbers represent the same value throughout the discussion. *Those literal numbers that may vary in a given problem are called* **variables,** and *those literal numbers that are held fixed are called* **constants.**

EXAMPLE 8 Variables and constants

(a) The resistance of an electric resistor is R. The current I in the resistor equals the voltage V divided by R, written as $I = V/R$. For this resistor, I and V may take on various values, and R is fixed. This means I and V are variables and R is a constant. For a *different* resistor, the value of R may differ.

(b) The fixed cost for a calculator manufacturer to operate a certain plant is b dollars per day, and it costs a dollars to produce each calculator. The total daily cost C to produce n calculators is

$$C = an + b$$

Here, C and n are variables, and a and b are constants, and the product of a and n is shown as an. For *another* plant, the values of a and b would probably differ.

If specific numerical values of a and b are known, say $a = \$7$ per calculator and $b = \$3000$, then $C = 7a + 3000$. Thus, constants may be numerical or literal. ■

EXERCISES 1.1

In Exercises 1–4, make the given changes in the indicated examples of this section, and then answer the given questions.

1. In the first line of Example 1, change the 5 to −3 and the −19 to 14. What other changes must then be made in the first paragraph?

2. In Example 4, change the 6 to −6. What other changes must then be made in the first paragraph?

3. In the left figure of Example 5, change the 2 to −6. What other changes must then be made?

4. In Example 6, change the $\frac{2}{3}$ to $\frac{3}{2}$. What other changes must then be made?

In Exercises 5 and 6, designate each of the given numbers as being an integer, rational, irrational, real, or imaginary. (More than one designation may be correct.)

5. $3, \quad \sqrt{-4}, \quad -\frac{\pi}{6}$

6. $-\sqrt{-6}, \quad -2.33, \quad \frac{\sqrt{7}}{3}$

In Exercises 7 and 8, find the absolute value of each number.

7. $3, \quad -4, \quad -\frac{\pi}{2}$

8. $-0.857, \quad \sqrt{2}, \quad -\frac{19}{4}$

In Exercises 9–16, insert the correct sign of inequality (> or <) between the given numbers.

9. $6 \qquad 8$

10. $7 \qquad 5$

11. $\pi \qquad -3.2$

12. $-4 \qquad 0$

13. $-4 \qquad -|-3|$

14. $-\sqrt{2} \qquad -1.42$

15. $-\frac{1}{3} \qquad -\frac{1}{2}$

16. $-0.6 \qquad 0.2$

In Exercises 17 and 18, find the reciprocal of each number.

17. $3, \quad -\frac{4}{\sqrt{3}}, \quad \frac{y}{b}$

18. $-\frac{1}{3}, \quad -0.25, \quad x$

In Exercises 19 and 20, locate each number on a number line, as in Fig. 1.3.

19. $2.5, \quad -\frac{12}{5}, \quad \sqrt{3}$

20. $-\frac{\sqrt{2}}{2}, \quad 2\pi, \quad \frac{123}{19}$

In Exercises 21–44, solve the given problems. Refer to Fig. 1.9 for units of measurement and their symbols.

21. Is an absolute value always positive? Explain.

22. Is 2.17 rational? Explain.

23. What is the reciprocal of the reciprocal of any positive or negative number?

24. Find a rational number between −0.9 and −1.0 that can be written with a denominator of 11 and an integer in the numerator.

25. Find a rational number between 0.13 and 0.14 that can be written with a numerator of 3 and an integer in the denominator.

26. If $b > a$ and $a > 0$, is $|b - a| < |b| - |a|$?

27. List the following numbers in numerical order, starting with the smallest: $-1, 9, \pi, \sqrt{5}, |-8|, -|-3|, -3.5$.

28. List the following numbers in numerical order, starting with the smallest: $\frac{-1}{5}, -\sqrt{10}, -|-6|, -4, 0.25, |-\pi|$.

29. If a and b are positive integers and $b > a$, what type of number is represented by the following?

(a) $b - a$ **(b)** $a - b$ **(c)** $\frac{b - a}{b + a}$

30. If a and b represent positive integers, what kind of number is represented by (a) $a + b$, (b) a/b, and (c) $a \times b$?

31. For any positive or negative integer: (a) Is its absolute value always an integer? (b) Is its reciprocal always a rational number?

32. For any positive or negative rational number: (a) Is its absolute value always a rational number? (b) Is its reciprocal always a rational number?

33. Describe the location of a number x on the number line when (a) $x > 0$ and (b) $x < -4$.

34. Describe the location of a number x on the number line when (a) $|x| < 1$ and (b) $|x| > 2$.

35. For a number $x > 1$, describe the location on the number line of the reciprocal of x.

36. For a number $x < 0$, describe the location on the number line of the number with a value of $|x|$.

37. A *complex number* is defined as $a + bj$, where a and b are real numbers and $j = \sqrt{-1}$. For what values of a and b is the complex number $a + bj$ a real number? (All real numbers and all imaginary numbers are also complex numbers.)

38. A sensitive gauge measures the total weight w of a container and the water that forms in it as vapor condenses. It is found that $w = c\sqrt{0.1t + 1}$, where c is the weight of the container and t is the time of condensation. Identify the variables and constants.

39. In an electric circuit, the reciprocal of the total capacitance of two capacitors in series is the sum of the reciprocals of the capacitances. Find the total capacitance of two capacitances of 0.0040 F and 0.0010 F connected in series.

40. Alternating-current (ac) voltages change rapidly between positive and negative values. If a voltage of 100 V changes to −200 V, which is greater in absolute value?

41. The memory of a certain computer has a bits in each byte. Express the number N of bits in n kilobytes in an equation. (A *bit* is a single digit, and bits are grouped in *bytes* in order to represent special characters. Generally, there are 8 bits per byte. If necessary, see Fig. 1.10 for the meaning of *kilo*.)

42. The computer design of the base of a truss is x m long. Later it is redesigned and shortened by y cm. Give an equation for the length L, in centimetres, of the base in the second design.

43. In a laboratory report, a student wrote "$-20°\text{C} > -30°\text{C}$." Is this statement correct? Explain.

44. After 5 s, the pressure on a valve is less than 600 kPa. Using t to represent time and p to represent pressure, this statement can be written "for $t > 5$ s, $p < 600$ kPa." In this way, write the statement "when the current I in a circuit is less than 4 A, the voltage V is greater than 12 V."

Answers to Practice Exercises

1. 4.2 **2.** $-\frac{3}{4}$ **3.** $<$ **4.** $>$

1.2 Fundamental Operations of Algebra

Fundamental Laws of Algebra • Operations on Positive and Negative Numbers • Order of Operations • Operations with Zero

The Commutative and Associative Laws

If two numbers are added, it does not matter in which order they are added. (For example, $5 + 3 = 8$ and $3 + 5 = 8$, or $5 + 3 = 3 + 5$.) This statement, generalized and accepted as being correct for all possible combinations of numbers being added, is called the **commutative law** for addition. It states that *the sum of two numbers is the same, regardless of the order in which they are added*. We make no attempt to prove this law in general, but accept that it is true.

In the same way, we have the **associative law** for addition, which states that *the sum of three or more numbers is the same, regardless of the way in which they are grouped for addition*. For example, $3 + (5 + 6) = (3 + 5) + 6$.

The laws just stated for addition are also true for multiplication. Therefore, *the product of two numbers is the same, regardless of the order in which they are multiplied*, and *the product of three or more numbers is the same, regardless of the way in which they are grouped for multiplication*. For example, $2 \times 5 = 5 \times 2$, and $5 \times (4 \times 2) = (5 \times 4) \times 2$.

The Distributive Law

Another very important law is the **distributive law.** It states that *the product of one number and the sum of two or more other numbers is equal to the sum of the products of the first number and each of the other numbers of the sum*. For example,

$$5(4 + 2) = 5 \times 4 + 5 \times 2$$

In this case, it can be seen that the total is 30 on each side.

■ Note carefully the difference:
associative law: $5 \times (4 \times 2)$
distributive law: $5 \times (4 + 2)$

In practice, these **fundamental laws of algebra** are used naturally without thinking about them, except perhaps for the distributive law.

Not all operations are commutative and associative. For example, division is not commutative, since the order of division of two numbers does matter. For instance, $\frac{6}{5} \neq \frac{5}{6}$ ($\neq$ is read "does not equal"). (Also, see Exercise 50.)

Using literal numbers, the fundamental laws of algebra are as follows:

Commutative law of addition: $a + b = b + a$

Associative law of addition: $a + (b + c) = (a + b) + c$

Commutative law of multiplication: $ab = ba$

Associative law of multiplication: $a(bc) = (ab)c$

Distributive law: $a(b + c) = ab + ac$

Each of these laws is an example of an *identity,* in that the expression to the left of the $=$ sign equals the expression to the right for any value of each of a, b, and c.

OPERATIONS ON POSITIVE AND NEGATIVE NUMBERS

When using the basic operations (addition, subtraction, multiplication, division) on positive and negative numbers, we determine the result to be either positive or negative according to the following rules.

Addition of two numbers of the same sign *Add their absolute values and assign the sum their common sign.*

EXAMPLE 1 Adding numbers of the same sign

(a) $2 + 6 = 8$ the sum of two positive numbers is positive

(b) $-2 + (-6) = -(2 + 6) = -8$ the sum of two negative numbers is negative

The negative number -6 is placed in parentheses since it is also preceded by a plus sign showing addition. It is not necessary to place the -2 in parentheses. ■

■ From Section 1.1, we recall that a positive number is preceded by no sign. Therefore, in using these rules, we show the "sign" of a positive number by simply writing the number itself.

Addition of two numbers of different signs *Subtract the number of smaller absolute value from the number of larger absolute value and assign to the result the sign of the number of larger absolute value.* Alternatively, one can visualize addition using the number line concept discussed in Section 1.1. Start with the number line location of the first number in the addition problem. Then, if you add a positive number, move *right* along the number line to the total. If you add a negative number, move *left* along the number line until you arrive at the solution.

EXAMPLE 2 Adding numbers of different signs

(a) $2 + (-6) = -(6 - 2) = -4$

(b) $-6 + 2 = -(6 - 2) = -4$

the negative 6 has the larger absolute value

(c) $6 + (-2) = 6 - 2 = 4$

(d) $-2 + 6 = 6 - 2 = 4$

the positive 6 has the larger absolute value

the subtraction of absolute values ■

Subtraction of one number from another *Change the sign of the number being subtracted and change the subtraction to addition. Perform the addition.*

EXAMPLE 3 Subtracting positive and negative numbers

(a) $2 - 6 = 2 + (-6) = -(6 - 2) = -4$

Note that after changing the subtraction to addition, and changing the sign of 6 to make it -6, we have precisely the same illustration as Example 2(a).

(b) $-2 - 6 = -2 + (-6) = -(2 + 6) = -8$

Note that after changing the subtraction to addition, and changing the sign of 6 to make it -6, we have precisely the same illustration as Example 1(b).

(c) $-a - (-a) = -a + a = 0$

Subtraction of a Negative Number

This shows that subtracting a number from itself results in zero, even if the number is negative. Therefore, *subtracting a negative number is equivalent to adding a positive number of the same absolute value.* ■

Multiplication and division of two numbers *The product (or quotient) of two numbers of the same sign is positive. The product (or quotient) of two numbers of different signs is negative.*

EXAMPLE 4 Multiplying and dividing positive and negative numbers

(a) $3(12) = 3 \times 12 = 36$ $\qquad \dfrac{12}{3} = 4$ result is positive if both numbers are positive

(b) $-3(-12) = 3 \times 12 = 36$ $\qquad \dfrac{-12}{-3} = 4$ result is positive if both numbers are negative

(c) $3(-12) = -(3 \times 12) = -36$ $\qquad \dfrac{-12}{3} = -\dfrac{12}{3} = -4$ result is negative if one number is positive and the other is negative

(d) $-3(12) = -(3 \times 12) = -36$ $\qquad \dfrac{12}{-3} = -\dfrac{12}{3} = -4$ ■

ORDER OF OPERATIONS

When mathematical operation symbols separate a series of numbers in an expression, it is important to follow an unambiguous *order* for completing those operations.

Order of Operations

1. Perform operations within specific groupings first—that is, inside parentheses ( ), brackets [ ], or absolute values | |.
2. Exponents and roots/radicals are evaluated next. **These will be discussed in Section 1.4 and Section 1.6, respectively.**
3. Perform multiplications and divisions (from left to right).
4. Perform additions and subtractions (from left to right).

EXAMPLE 5 Order of operations

(a) $20 \div (2 + 3)$ is evaluated by first adding $2 + 3$ and then dividing. The grouping of $2 + 3$ is clearly shown by the parentheses. Therefore, $20 \div (2 + 3) = 20 \div 5 = 4$.

(b) $20 \div 2 + 3$ is evaluated by first dividing 20 by 2 and then adding. No specific grouping is shown, and therefore the division is done before the addition. This means $20 \div 2 + 3 = 10 + 3 = 13$.

(c) $16 - 2 \times 3$ is evaluated by ***first multiplying*** 2 ***by*** 3 and then subtracting. We *do* **not** *first subtract* 2 *from* 16. Therefore, $16 - 2 \times 3 = 16 - 6 = 10$.

(d) $16 \div 2 \times 4$ is evaluated by first dividing 16 by 2 and then multiplying. From left to right, the division occurs first. Therefore, $16 \div 2 \times 4 = 8 \times 4 = 32$.

(e) $|3 - 5| - |-3 - 6|$ is evaluated by first performing the subtractions within the absolute value vertical bars, then evaluating the absolute values, and then subtracting. This means that $|3 - 5| - |-3 - 6| = |-2| - |-9| = 2 - 9 = -7$. ■

■ Note that $20 \div (2 + 3) = \frac{20}{2 + 3}$, whereas $20 \div 2 + 3 = \frac{20}{2} + 3$.

Practice Exercises

Evaluate: **1.** $12 - 6 \div 2$
2. $16 \div (2 \times 4)$

COMMON ERROR

Remember that order of operations takes precedence over perceived left-to-right sequences of operators.

$20 + 10 \div 5$ is evaluated by *first dividing* 10 by 5, *then adding* the result to 20.

$20 + 10 \div 5 = 20 + 2 = 22$ is evaluated correctly.

A common error would be to perform the addition first:

$20 + 10 \div 5 \neq 30 \div 5 = 6$

When evaluating expressions, it is generally more convenient to change the operations and numbers so that the result is found by the addition and subtraction of positive numbers. When this is done, we must remember that

$$a + (-b) = a - b \tag{1.1}$$

$$a - (-b) = a + b \tag{1.2}$$

Practice Exercises

Evaluate: **3.** $2(-3) - \frac{4-8}{2}$

4. $\frac{|5-15|}{2} - \frac{-9}{3}$

EXAMPLE 6 Evaluating numerical expressions

(a) $7 + (-3) - 6 = 7 - 3 - 6 = 4 - 6 = -2$ using Eq. (1.1)

(b) $\frac{18}{-6} + 5 - (-2)(3) = -3 + 5 - (-6) = 2 + 6 = 8$ using Eq. (1.2)

(c) $\frac{|3-15|}{-2} - \frac{8}{4-6} = \frac{12}{-2} - \frac{8}{-2} = -6 - (-4) = -6 + 4 = -2$

(d) $\frac{-12}{2-8} + \frac{5-1}{2(-1)} = \frac{-12}{-6} + \frac{4}{-2} = 2 + (-2) = 2 - 2 = 0$

In illustration (b), we see that the division and multiplication were done before the addition and subtraction. In (c) and (d), we see that the groupings were evaluated first. Then we did the divisions, and finally the subtraction and addition.

EXAMPLE 7 Evaluating in an application

A 1500-kg van going at 40 km/h ran head-on into a 1000-kg car going at 20 km/h. An insurance investigator determined the velocity of the vehicles immediately after the collision from the following calculation. See Fig. 1.6.

$$\frac{1500(40) + (1000)(-20)}{1500 + 1000} = \frac{60\,000 + (-20\,000)}{1500 + 1000} = \frac{60\,000 - 20\,000}{2500}$$

$$= \frac{40\,000}{2500} = 16 \text{ km/h}$$

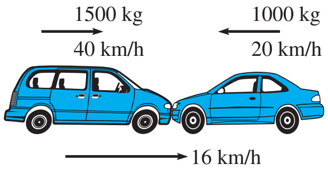

Fig. 1.6

The numerator and the denominator must be evaluated before the division is performed. The multiplications in the numerator are performed first, followed by the addition in the denominator and the subtraction in the numerator.

OPERATIONS WITH ZERO

Since operations with zero tend to cause some difficulty, we will show them here.

If a is a real number, the operations of addition, subtraction, multiplication, and division with zero are as follows:

$$\boldsymbol{a + 0 = a}$$

$$\boldsymbol{a - 0 = a} \qquad \boldsymbol{0 - a = -a}$$

$$\boldsymbol{a \times 0 = 0}$$

$$\boldsymbol{0 \div a = \frac{0}{a} = 0} \qquad (\textbf{if } \boldsymbol{a \neq 0})$$

($\neq$ means "is not equal to")

LEARNING TIP

Division by zero is undefined because no real value can be associated with that division.

If $c = \frac{4}{0}$, then $c \times 0 = 4$, which is *not true*, since $c \times 0 = 0$ for any value of c.

There is a special case of division by zero termed *indeterminate* because no specific value can be determined from the division, but many real values are indeed possible.

If $c = \frac{0}{0}$, then $c \times 0 = 0$, which is true, since $c \times 0 = 0$ for *any* value of c.

Thus, $c = 13$, $c = -4.76$, and $c = 0$ are *all* valid solutions for the division.

EXAMPLE 8 Operations with zero

(a) $5 + 0 = 5$ **(b)** $-6 - 0 = -6$ **(c)** $0 - 4 = -4$

(d) $\frac{0}{6} = 0$ **(e)** $\frac{0}{-3} = 0$ **(f)** $\frac{5 \times 0}{7} = \frac{0}{7} = 0$

Note that there is no result defined for division by zero. To understand the reason for this, consider the results for $\frac{6}{2}$ and $\frac{6}{0}$.

$$\frac{6}{2} = 3 \quad \text{since} \quad 2 \times 3 = 6$$

If $\frac{6}{0} = b$, then $0 \times b = 6$. This cannot be true because $0 \times b = 0$ for any value of b.